DESIGN FOR INJECTION MOLDING:
BALANCING MECHANICAL REQUIREMENTS, MANUFACTURING COSTS, AND MATERIAL SELECTION

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ABSTRACT

This paper describes a procedure for considering mechanical requirements, manufacturing costs, and material selection in the design of injection molded parts. The benefit of this approach is the simultaneous consideration of the implications of material selection and part geometry on estimated manufacturing cost during candidate design selection. The current implementation uses the allowable deflection of a flat plate as an example performance measure. Manufacturability concerns include required part thickness and gating scheme to adequately mold the part, and a resulting cycle time based on part cooling time estimates. Part manufacturing cost includes material cost, cycle time, and production costs. A PC-based and CAD-integrated program illustrate our proposed procedure.

1. INTRODUCTION

1.1 Background

Injection molding is a powerful and comprehensive manufacturing process. However, it can also be a complicated and costly experience. The designer must not only consider part performance requirements, but process and material constraints as well. Effective management of these constraints in a time-efficient manner can yield significant product cost savings and a quick time-to-market.

Much savings can be realized by making informed decisions early in the design process. Currently, the designer has several tools and forms of knowledge at his disposal. In the early stages of design, and throughout much of the design process, the designer can employ qualitative knowledge in the form of personal experience and tabulated rules of thumb. After the design has been detailed, the designer can use FEM to assess mechanical and thermal performance, and process simulation to reveal processing trouble-spots with the design.

Unfortunately, FEM and process simulation require some geometric, process, and material information which only analysts can use effectively. The need for this specialized information is understandable, since the polymer process numerical model is complex. Thus, this level of detail is usually considered only well into the design process, at which point the designer is less willing to make the changes that the simulation might suggest. We feel that the designer needs process models and knowledge that fill in the gap between rules-of-thumb and process simulation. We acknowledge that the less detail known about a design, the less complex and accurate any process model can be. However, we feel much can be gained from performing analytical and experimental studies, and then providing the designer with “distilled” models that will provide levels of accuracy and complexity commensurate with the level of design information.

1.2 Previous Work

Much previous work has been performed in the area of injection mold filling, packing and cooling simulation (Chu, et al., 1989; Tanguy, et al., 1988; Wang, et al., 1986), and has led to many software packages that perform this analysis, with varying degrees of success (Austin, 1990; Nedeb, et al., 1992; Weissmann, 1991). Recently, focus has been placed on the behavior of the plastic after ejection from the mold (Kabanemi, et al., 1992; Rezayat, 1989; Santhanam, et al., 1992). This has centered on part shrinkage and warpage due to thermal- and flow-induced strain. Validation studies of the shrinkage and warpage predictions are now published (DiScipio, et al., 1990; Post, 1991), and it is obvious that this is a much more complex problem than filling and cooling analysis.

Other approaches to DFIM incorporate heuristic knowledge about good and bad plastic part design (Ishii, et al., 1989). The goal of this knowledge-based approach is to assist the plastic part designer during the upstream design phase, before final decisions have been made regarding geometry, material, and processing parameters. This approach was extended by performing experiments examining part quality and incorporating the results in the knowledge-based system as an empirical model (Beiter, et al., 1991; Mehl, et al., 1994).

Of critical importance to good design practice is material selection. With ever increasing amounts of plastics available to meet specialized needs, this subject has received increased attention. Beiter, et al. (1991) describe a procedure for selecting a polymer based on a weighted sum of material property preferences. Oehler, et al. (1994) discuss integrating basic requirements (UV resistance, flammability rating), loading, and manufacturing constraints into material selection. Ashby (1992) describes a procedure for material selection based on an objective function for design attributes such as weight, cost, strength and stiffness. The objective function characterizes the functional requirements, geometry, and material properties and, combined with specified constraint precedence, leads to a *material performance index*, which is used to select a material.

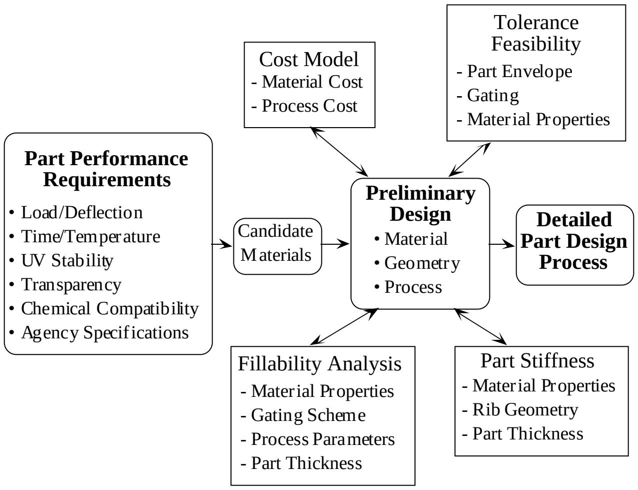

Figure 1: Computer program schematic

1.3 Our Approach

This current project seeks to bridge the gap between process simulation and knowledge-based approaches by adapting previous research on material selection, part fillability, and stress analysis into a comprehensive approach and computer program to evaluate candidate material and part envelope feasibility. Specifically, this paper will address:

1) Material Selection
2) Part Fillability
3) Part Stiffness
4) Cooling Time
5) Preliminary Part Manufacturing Cost

The end result of this research is a computer decision support tool that facilitates the selection of candidate materials that meet basic processing, functional, and economic requirements (Figure 1). The following sections explain the approach in more detail.

2. PRELIMINARY PLASTIC PART DESIGN

2.1 Overview

The geometric decision variable that has the largest impact on mechanical performance is wall thickness. Since wall thickness is so strongly coupled with processibility, any design technique must also consider material and process constraints on wall thickness configuration. There are a number of factors that influence the choice of wall thickness, including:

Mechanical Requirements: Nominal wall thickness must meet basic part requirements such as static, dynamic, and impact loading.

Material and Process: Higher-flow materials will permit thinner wall designs, while thinner wall designs will in general require higher injection pressures.

Production rate: Thicker parts will require longer cooling times and increase the cycle time.

Part Geometry: Assembly and functional surface protrusions will require variations in the nominal wall. These may cause uneven cooling, part distortion, and residual stresses.

Agency Requirements: Electrical and flammability considerations place certain restrictions on part thickness.

From a design perspective, the part mechanical performance will place certain (and very important) restrictions on wall thickness and part geometry. In many engineering applications, such mechanical performance concerns include strength & stiffness, impact strength, fatigue, and creep and stress relaxation. Good engineering practice suggests that the designer optimize the material, geometry, and process design to meet the mechanical, process, and production constraints to the extent possible.

2.2 Mechanical Constraints

2.2.1 Strength and Stiffness. In many cases, structural geometry can be approximately modeled by a flat plate with prescribed loadings and boundary conditions. Using classical plate theory, many solutions exist for distributed and point loadings with free, simple and clamped edge conditions (Timoshenko et al, 1959; Reismann, 1988; Young, 1989). Classical Plate Theory (CPT) makes the following assumptions (Young, 1989):

- The plate is flat and of uniform thickness and material.
- The plate thickness is less than one-quarter the length of the shortest side.
- The maximum deflection is not more than about one-half the plate thickness.
- All forces are normal to the plane of the plate.
- All stresses are within the elastic limit.

As an example, consider a flat plate with clamped edges under a uniformly distributed load (Figure 2).

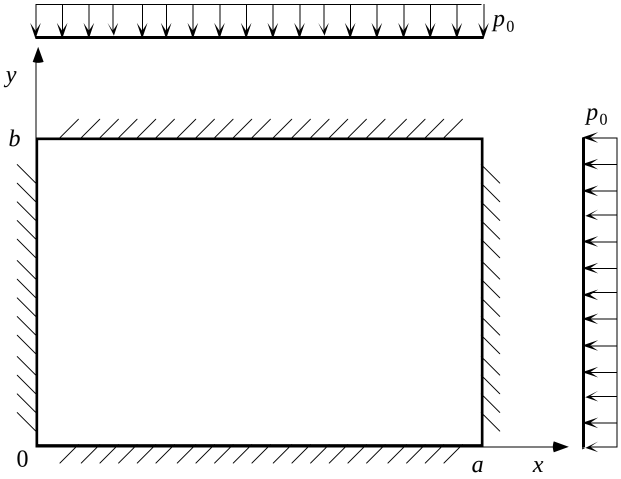

Figure 2: Uniform loading of a flat plate with clamped edges

Maximum deflection, $w_{\max}$, and maximum stress, $\sigma_{\max}$, of the plate shown in Figure 2 is given by (Young, 1989):

$$w_{\max} = w\left(\frac{a}{2},\frac{b}{2}\right) = \frac{\alpha p_0 b^4}{Eh^3} \tag{1}$$

$$\sigma_{\max} = \sigma\left(\frac{a}{2},\pm\frac{b}{2}\right) = \frac{-\beta_1 p_0 b^2}{h^2} \tag{2}$$

where

p_0 = uniform load
α = constant determined by a/b in Table 1
β_1 = constant determined by a/b in Table 1
h = plate thickness
E = modulus of elasticity
υ = poisson's ratio = 0.3

Table 1: Constants α and β_1 for various values of a/b (Young, 1989)

a/b	1.0	1.2	1.4	1.6	1.8	2.0	∞
α	0.0138	0.0188	0.0226	0.0251	0.0267	0.0277	0.0284
β_1	0.3078	0.3834	0.4356	0.4680	0.4872	0.4974	0.5000

Interpolation between deflections and stress values provided for given ratios of a/b will give reasonable estimates for a/b ratios not listed in Table 1. Note that CPT does not include time and temperature effects on stiffness (creep and stress relaxation), which are often important when designing thermoplastic engineering components.

2.2.2 Creep. In nearly all structural applications with engineering thermoplastics, designers must consider the effects of time, temperature, and stress level. Many studies have examined the behavior of test specimens under various loading conditions over time, and large volumes of data are available in the form of time-strain plots at various temperatures (Trantina, et al., 1994).

For computational purposes, a second-order polynomial function in log time has exhibited reasonable correlation (Trantina, et al., 1994):

$$\varepsilon_t = A(\log t)^2 + B\log t + C \tag{3}$$

where

ε_t = creep strain (%)
t = time (seconds)
A, B, C = material constants

To capture the effects of temperature, interpolation and extrapolation in natural log strain–inverse temperature space is adequate (Trantina, et al., 1994); i.e., solve

$$\ln \varepsilon_c = \frac{C_1}{T} + C_2 \tag{4}$$

for C_1 and C_2 at two nearby temperatures in conjunction with creep strains determined with Equation 3, and use Equation 4 to solve for the unknown creep strain at the new temperature. In Equation 4

ε_c = creep strain (%)
T = absolute temperature (°)

For including time and temperature effects in a constant stress application, designers can use an *effective modulus* in the standard plate equations (e.g., Equation 1). The effective modulus $E(t,T)$ is defined as

$$E(t,T) = \frac{\sigma_0}{\varepsilon_c(t,T)} \tag{5}$$

where

σ_0 = constant initial stress
ε_c = creep strain

Selection of the appropriate effective modulus requires engineering judgment, since the stress-time data is recorded at specific applied stress levels.

2.2.3 Impact. In many applications, dynamic impact loading of a part may induce failure. Plastics are often chosen for their ability to absorb high amounts of energy before permanent deformation. However, proper material and geometry selection is required to minimize the chance of failure under impact loading. Consider, for example, a ribbed plate subjected to a known impact load P (Figure 3). Woods, et al. (1995) have shown that the ratio of maximum principal stress, σ_1, to yield stress, σ_y, as a function of strain rate is useful for ensuring *ductile* failure, which is a more desirable mode of failure than catastrophic *brittle* failure. They relate this stress ratio to plate thickness and rib fillet radius (Figure 4). By ensuring that the rib radius to plate thickness ratio falls to the right of the region of brittle failure, defined by the line of constant σ_1/σ_y, the part will fail in a ductile manner. However, in some situations prescribing a larger fillet radius will induce *sink mark* in the region under the rib (indicated in the region labeled “processing problems” in Figure 4). In such a case, the designer will have to change the geometry, or rely on more costly processing conditions to mold satisfactory parts.

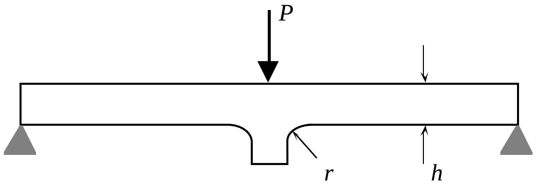

Figure 3: Flat plate with rib under impact load *P*

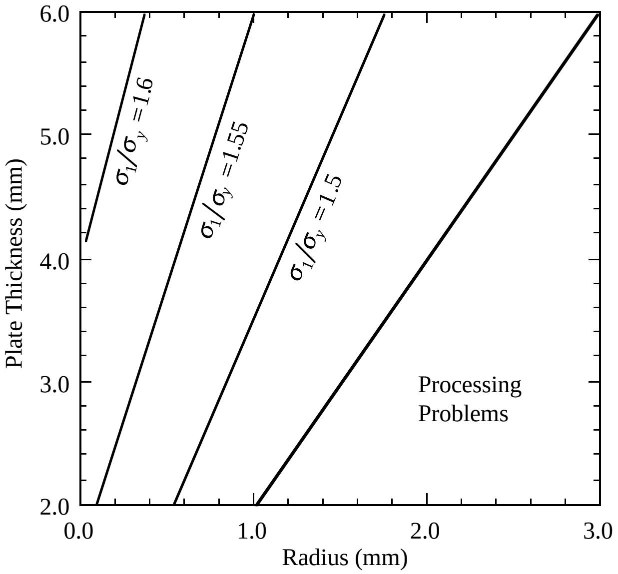

Figure 4: Maximum principal stress to yield stress as a function of fillet radius *r* and plate thickness *h*

2.3 Material and Process Constraints

2.3.1 Material Candidate Pruning. Material selection is a important step that is made early in the design process. Corporate agreements or carry-over designs often mandate that a certain material be used from the outset of the design process; in these cases, material selection is not a design-based decision. However, the selection of a material is a critical design decision which should not be fixed before a material's compatibility with the other aspects of design is evaluated.

A potential problem that often arises is how to ensure the compatibility of all the potential materials with the requirements of a part design. However, even before any geometry or processing information is prescribed, the design team is aware of certain requirements on and environments in which the design is intended to perform. Using these application constraints, the list of candidate materials can be greatly reduced. For example, in the design of a part for an underhood automotive application, the design team could immediately eliminate material families such as ABS or polyethylene due to their incompatibility with the intended application of the part. We refer to the reduction of candidate materials as material candidate pruning (MCP). The MCP process will usually leave only a handful of material families which are compatible with the intended application of the part. Using a pruned list, members of the design team can perform design analyses using only the materials in this manageable list.

2.3.2 Part Fillability. Wall thickness is a major concern in the moldability of a thermoplastic part. Typically, the nominal wall thickness is specified after the general shape of the part is determined and before the details of the features are determined. There are general guidelines for determining nominal wall thickness, but this is difficult to do without knowledge of gating scheme, process window, structural requirements, and material. The gating scheme will alter the maximum flow length of the material, the material will determine the viscosity and the achievable flow length, and the process window will tell the designer how much flow length is achievable by variations in the process window. Designers should consider the gating scheme, process window, and material at the stage when it is appropriate to determine wall thickness.

In this paper, fillability analysis refers to the part design methodology that facilitates the determination of the minimum wall-thickness required in order to fill a part design for a combination of material, process, and part geometry. The use of this methodology allows designers to explore a broader range of solutions before the designer has committed to any particular design. To perform our fillability analyses, we will use a fillability model (Mehl, et al., 1994) to relate the thickness of a part design with maximum flow length during processing for a specific gating scheme, type of material, and set of process conditions. Using the empirical relation in the fillability model, the user can either determine a minimum thickness based on a defined flow length or vice versa. During the analysis, the user can change the gating scheme, material, and/or process settings and repeat the fillability calculations. With these results, the designer can assess the feasible solutions and arrive at a manufacturable part design. We envision that the designer will complete the fillability analysis in two steps.

Initial Fill Area Analysis: the goal of this analysis is to use the fillability model and basic design information to estimate the minimum number of gates required to fill a part. To perform this first cut analysis, the designer must provide the system with a selected material from the pruned list of candidate materials and some machine information including the barrel diameter and ram speed. The user has the option to provide a specific processing temperature or nominal thickness. If values are not provided, nominal values can be retrieved from the database. The final input for this analysis is the unfolded 2-D fill area, A_{fill}, which can be approximated from a CAD model by using a thin-wall assumption.

The determination of the minimum number of gates uses an iterative technique. Following the fillability model application techniques (Mehl, et al., 1994), the system will use a starting flow length, fl, to determine the average melt front width, average velocity and shear rate. Using these values, the system will determine the temperature and shear rate dependent viscosity which will be used to calculate the fillability number:

$$FN = \log\left[\frac{\eta/\rho}{Vh}\right] \tag{6}$$

where

FN = Fillability Number
ρ = density
η = viscosity
V = velocity
h = thickness

Using the fillability number and the material specific fillability constants (C and k_c), the system can determine the new maximum value for the flow length:

$$fl = h^2 C \cdot 10^{k_c \cdot FN} \tag{7}$$

where

fl = flow length
t = thickness
C = fillability constant
k_c = fillability constant
FN = Fillability Number

The iterations will continue until the change in the maximum flow length is small. The minimum number of gates is then found by :

$$N_g = \frac{A_{fill}}{\pi fl^2} \tag{8}$$

where

N_g = minimum number of gates
A_{fill} = area of fill
fl = flow length

The number of gates should be rounded up to the nearest whole number.

Whole Part Fillability Analysis: using the recommendation for the minimum number of gates, the user can finish the fillability analysis. This step will determine if the part will fill based on gate locations and geometric complexities. Most of the necessary inputs will come from the initial fill area analysis. The final input from the user is the placement of the gates on the part geometry. Using the gate locations, the system will evaluate the maximum flow length based on the geometry and compare it to the maximum flow length from the fillability model. If the maximum flow length in the part geometry does not exceed the flow length from the fillability model, then the design system will return an affirmation of the design; otherwise it will give a warning and ask that the designer retry gating placement to attain part filling.

2.3.3 Cycle Time.

Cycle time is the time required to mold one part (in a single cavity mold) and consists of mold closing time, injection time, cooling time, mold opening time, and part ejection time. In general, the cooling time is the largest component of cycle time. While process simulation can provide good estimates of required cooling time, there are approximate methods available for rough geometry. Equation 9 gives the estimated cooling time for a flat plate as shown in Figure 5 (Malloy, 1994). Again, wall thickness, h, has a large effect on cooling time.

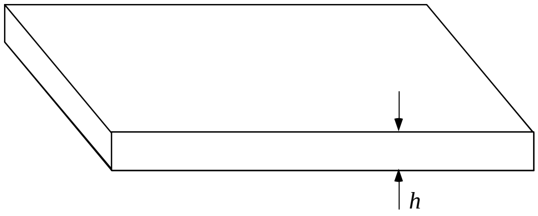

Figure 5: Flat plate geometry for cooling calculations

$$t_c = \frac{h^2}{\alpha\pi^2}\ln\left[\frac{4}{\pi}\left(\frac{T_m - T_w}{T_e - T_w}\right)\right] \tag{9}$$

where

t_c = time for centerline to reach T_e (s)
h = plate thickness (m)
$\alpha = k/\rho c$: thermal diffusivity $(\mathrm{m^2/s})$
T_m = melt temperature (°)
T = ejection temperature (°)
T_w = mold temperature (°)
k = thermal conductivity (W/m°K)
c = specific heat (J/kg°K)
ρ = density $(\mathrm{kg/m^3})$

Table 2 provides typical values of some selected polymers, and Figure 6 shows cooling time as a function of thickness using Equation 9.

Table 2: Material Data used for Cooling Curve Calculations

Mat.	k $\left(\frac{\mathrm{W}}{\mathrm{m \cdot K}}\right)$	c $\left(\frac{\mathrm{J}}{\mathrm{kg \cdot K}}\right)$	ρ $\left(\frac{\mathrm{kg}}{\mathrm{m^3}}\right)$	T_m (°F)	T_w (°F)	T_e (°F)
PC	0.190	1298	1200	575	180	245
ABS	0.264	1314	1040	460	135	180
PBT	0.264	1741	1310	470	105	240
PC/ABS	0.246	1252	1120	500	175	200

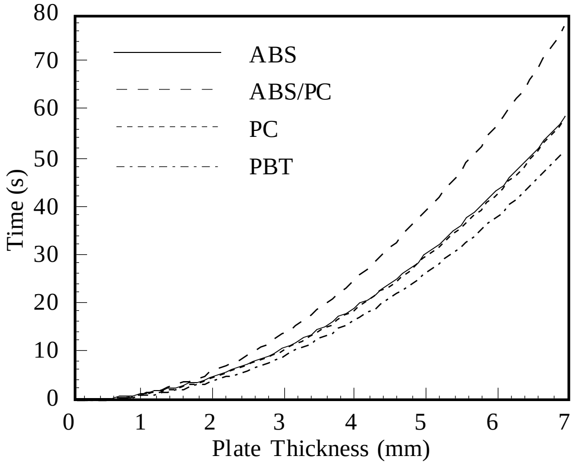

Figure 6: Cooling time vs. plate thickness

3. EVALUATING PART COST

3.1 Cost Model

Our cost model is based on an estimated cycle time (from the cooling time calculation) and per part cost based on material cost and part volume:

$$C_T = \underbrace{\frac{\left(C_{mach} + C_{labor}\right)}{3600} t_c}_{\text{Processing Cost}} + \underbrace{V \cdot \rho \cdot C_{mat}}_{\text{Part Cost}} \tag{10}$$

where

$C_T =$ total cost of the part $(\$/\text{part})$

$t_c =$ cycle time for each part (s/part)

$C_{mach} =$ injection molding machine cost $(\$/\text{hr})$

$C_{labor} =$ labor cost $(\$/\text{hr})$

$V =$ volume of the part $\left(\text{m}^3/\text{part}\right)$

$\rho =$ density of the material $\left(\text{kg}/\text{m}^3\right)$

$C_{mat} =$ cost of the material $(\$/\text{kg})$

The cost model given by Equation 10 is an approximation of variable cost per part, and does not include mold construction costs, mold complexity (e.g., sliders), number of mold cavities, total production run, components of part cycle time due to injection and mold open and closing time, retooling and setup time, colorant, and scrap. In a detailed cost analysis, all of these factors should be considered. For purposes of comparing costs between different candidate part designs, we feel that this estimate is adequate; at such an early stage in the design process, details of the mold construction regarding complexity and number of cavities is not usually known.

3.2 System Structure

Figure 7 shows graphically the relationships between material and geometry choice and part cost. Our proposed system considers the effect of material and wall thickness choice on stiffness, part fillability, and required cooling time. The part cost is then computed for various material choices and part thicknesses that meet the mechanical and manufacturing requirements.

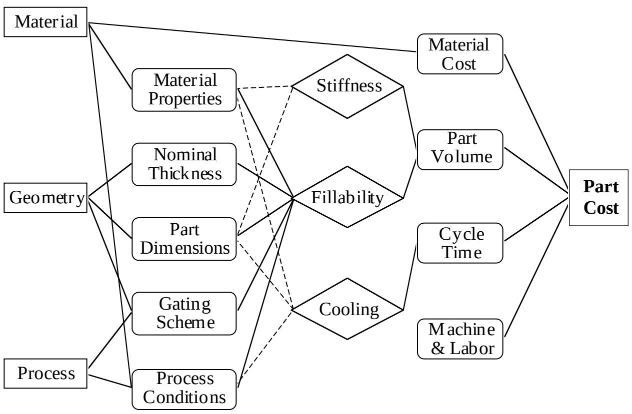

Figure 7: System schematic

4. COMPUTERIZED TOOL

4.1 PC-based Implementation

The end result of this research is a computer decision support tool that facilitates the selection of candidate materials and rough geometry that meet basic processing, functional, and economic requirements.

Figure 8 shows a screen in the PC design aid for entering loading conditions, edge conditions, permissible deflection, application environmental temperature, and required time (for creep investigation).

Figure 8: Input screen for plate stiffness calculations

The calculation summary screen is shown in Figure 9. The PC tool produces a list of candidate materials ranked by normalized cost (relative to the lowest cost candidate) based on a desired maximum plate deflection δ. The resulting plate thickness permits calculation of a cooling time and achievable flow length. The current tool does not include fillability requirements in the final cost score, however, the program does note the gating requirements for each material (edge-gated, center-gated, or multiple drops required).

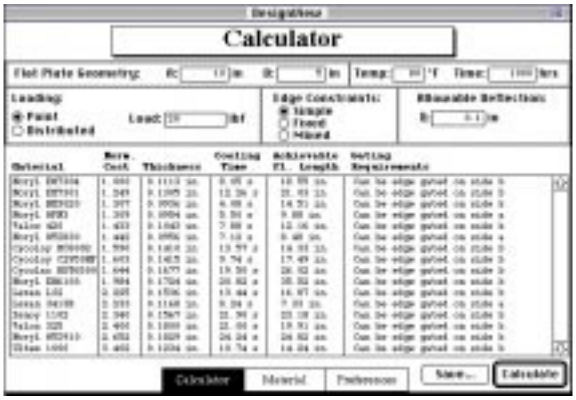

Figure 9: PC Output screen showing normalized cost

4.2 CAD Implementation

This section briefly describes the implementation of one of the methodologies mentioned above into a workstation-based CAD system, Pro/Engineer. The main difference between the CAD-integrated tool and the PC tool is that the CAD tool can leverage the geometric information provided by the CAD system. Accordingly, it does not require the user to re-enter part geometry information. Another major benefit of using CAD data is that more complex and generic geometries can be evaluated. However, using generalized geometry requires that the evaluation techniques be massaged to use the information (such as volume and surface area) that the CAD system can readily provide. We integrated our design system into Pro/Engineer using Pro/Develop. The use of the Pro/Develop interface enables us to seamlessly integrate our advisory application into the Pro/Engineer environment. The user will be able to trigger the manufacturability evaluation functions through menu selections and use them to gain feedback on the design.

One module of the Pro/Engineer - integrated design system is the fillability analysis. Recall that the fillability algorithm outlined in section 2.3.2 was divided into two steps: (1) estimation of gates and (2) final determination of fillability based on gate placement. We have included the first step of the fillability analysis as an example here. The user will engage the tool through an appropriate menu selection. The application provides for the user to enter material and processing information in a Motif environment (Figure 10). Note that the tool does not require that geometric information be explicitly entered by the user; it will be extracted from the geometric database.

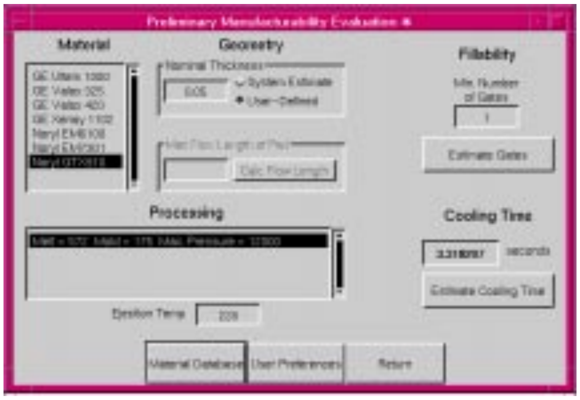

Figure 10: Pro/Engineer integrated costing tool

Upon entering the required information, the tool will perform the numerical analysis in section 2.3 to arrive at the maximum flow length that is achievable. Accordingly, it can estimate the minimum number of gates and display a summary upon request from the user.

This system provides a tool which will help designers evaluate manufacturability within an environment that designers are comfortable. Using the feedback from the manufacturability assessments, designers can make more manufacturable part designs and reduce design iterations.

5. CONCLUSIONS AND FUTURE WORK

In this paper the authors describe a procedure for simultaneously considering mechanical requirements, manufacturing costs, and material selection in the design of injection molded parts. The benefit of this approach is the consideration of the implications of material selection and part geometry on estimated manufacturing cost during candidate design selection.

The current implementation uses the allowable deflection of a flat plate as an example performance measure. It is assumed that a flat plate geometry can adequately approximate the candidate geometry. Deflections and maximum stresses are computed using classical plate theory, which are valid under the assumptions stated. Time and temperature behavior is also considered in the plate calculations. Impact and fatigue issues are mentioned in the paper, but are currently not included in the system.

Manufacturability concerns include required part thickness and gating scheme to adequately mold the part, and a resulting cycle time based on part cooling time estimates. Part fillability is estimated by using a non-dimensional fillability number, which the authors find adequate for estimation at the early stages of design. Of course, process simulation techniques can provide more accurate measures for both filling requirements and cooling times once more detailed geometry, process and material information is known.

Part manufacturing cost includes material cost, cycle time, and production costs. More detailed cost analyses are available, however the authors feel that the cost model proposed is appropriate for the preliminary design stage and useful for design candidate comparison.

In the future, the authors intend to investigate the inclusion of more complex geometry and mechanical requirements (impact and fatigue). A CAD environment may permit a more detailed analysis of stiffness, filling, and cooling requirements. The authors are actively investigating using currently available tools in a seamless CAD environment.

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REFERENCES

Ashby, M. F., *Materials Selection in Mechanical Design*, Pergamon Press, Oxford, England (1992)

Austin, C., "Verification of Flow Analysis Software: Theory and Practice," SPE ANTEC Proceedings, Volume 36 (1990), pg. 209

Beiter, K., Hornberger, L., and Ishii, K., "Geometric Index for Predicting Sink Mark in Plastic Parts," Proc. of the ASME Design Theory and Methodology Conf., Sept., 1991, Miami, FL, pp. 111-118.

Beiter, K., Krizan, S., Ishii, K., and Hornberger, L., "HyperQ/Plastics: An Intelligent Design Aid for Plastic Material Selection," *International Journal of Advances in Software Engineering*, 1993, Vol.16, pp. 53-60

Chu, E., Kamal, M. R., and Goyal, S. K., "A Computer Simulation of the Injection Molding Process Including Filling, Packing, and Solidification," SPE ANTEC Proceedings, Volume 35 (1989), pg. 344

DiScipio, W., Wagle, A., and McCarthy, S. P., "Verification Mold for Flow, Cooling, Shrinkage and Warpage Analysis," SPE ANTEC Proceedings, Volume 36 (1990), pg. 283

Ishii, K., Hornberger, L., and Liou, M., "Compatibility-Based Design For Injection Molding," Proceedings of the 89 ASME WAM, Dec, (1989)

Kabanemi, K. K., and Crochet, M. J., "Thermoviscoelastic Calculation of Residual Stresses and Residual Shapes of Injection Molded Parts," *International Polymer Processing*, Vol. 7, No. 1, pg. 60, (1992)

Malloy, R. A., *Plastic Part Design for Injection Molding*, Hanser Publishers, New York, NY (1994)

Mehl, D. C., Beiter, K. A., and Ishii, K., "Design for Injection Molding: Using Dimensional Analysis to Assess Fillability," Proc. of the 1994 ASME Design Automation Conference, Minneapolis, MN, September, 1994.

Nedeb, C., and Kosin, T., "Selecting Simulation Programs for Injection Molding," Kunststoffe German Plastics, Vol. 82, No. 6, pg. 11 (1992)

Oehler, P. R., Graichen, C. M., and Trantina, G. G., "Design-based Material Selection" SPE ANTEC Proceedings (1994), pp. 3092-3096

Post, S., "Verification of Warpage Simulation in Injection Molding," M.S. Thesis, Department of Mechanical Engineering, The Ohio State University, (1991)

Reismann, H., *Elastic Plates, Theory and Application*, John Wiley & Sons, New York, NY (1988)

Rezayat, M., "Numerical Computation of Cooling-Induced Residual Stress and Deformed Shape for Injection-Molded Thermoplastics," SPE ANTEC Proceedings, Volume 35 (1989), pg. 341

Rosato, D. V., and Rosato, D. V., *Injection Molding Handbook*, Van Nostrand Reinhold, New York (1986)

Rosen, D. W., Dixon, J. R., Poli, C., and Dong, X., "Features and Algorithms for Tooling Cost Evaluation in Injection Molding and Die Casting," Proc. of the 1992 ASME Computers in Engineering Conference, San Francisco, CA, August, 1992, pp. 45 -52.

Santhanam, N., Chang, H.H., Himasekhar, K., Tuschak, Peter, and Wang, K. K., "Postmolding and Load-Induced Deformation of Plastic Parts in the Injection Molding Process," *Advances in Polymer Technology*, Vol. 11, No. 2, pg. 77 (1991/1992)

Tanguy, P. A., Fortin, A., and Bertrand, F., "A 3-D Finite Element Model for Polymer Melt Flow," Advances in Polymer Technology, Vol. 8, No. 2, pg. 99 (1988)

Timoshenko, S., and Woinowsky-Kreiger, S., *Theory of Plates and Shells*, McGraw Hill, New York, NY (1959)

Trantina, G. G., and Nimmer, R., *Structural Analysis of Thermoplastic Components*, McGraw Hill, New York, NY (1994)

Wang, V. W., Hieber, C. A., and Wang, K. K., "Mold-Filling-Simulation in Injection Molding of Three-Dimensional Thin Parts," SPE ANTEC Proceedings, Volume 32 (1986), pg. 97

Weissmann, D., "Mold Filling Simulations--Do They Provide the Needed Answers?," SPE ANTEC Proceedings, Volume 37 (1991), pg. 264

Woods, J. T., and Nimmer, R. P., *Impact Design Guide*, GE Corporate Research and Development, Schenectady, NY (1995)

Young, W. C., *Roark's Formulas for Stress & Strain*, 6th Edition, McGraw Hill, New York, NY (1989)