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ROBUST DESIGN OPTIMIZATION USING SIMULATION AND FACTORIAL EXPERIMENTS

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ABSTRACT

This paper describes a robust optimization methodology for design involving either complex simulations or actual experiments. The proposed procedure optimizes the worst case response that consists of a weighted sum of expected mean and deviation index (DI). The estimation scheme for expected mean and DI adopts the modified 3-point Gauss quadrature integration to assure superior accuracy for systems with significant nonlinear effects. The procedure incorporates uncertainties in design variables and variations in constraints due to uncertainty in design variables. The three proposed methods to incorporate variations in constraints are based on: 1) heuristics, 2) built-in constraint variation, and 3) differentiating KKT optimality conditions. We apply the proposed methods to gears with minimum noise under manufacturing errors and heat treated parts with minimum post heat treatment machining.

1. INTRODUCTION

One of the important tasks in engineering design is to account for factors that are either impossible or very expensive to control: material properties, manufacturing errors, and operational variances, etc. Traditionally, engineers conducted sensitivity analysis after design optimization. Taguchi's (1978 and 1987) introduced the concept of *parameter design* to improve the quality of a product whose manufacturing process involves significant variability. The parameter design concept "reduces variation in performance by reducing the sensitivity of an engineering design to sources of variation rather than controlling the sources." d'Entremont and Ragsdell (1988) developed a non-linear code that applies Taguchi's concepts to design optimization. Their goal was to minimize performance variability when design is constrained to have target performance. Ideally, one should optimize the expected value and its variation of a performance function when there are uncertainties in design variables.

Also, satisfaction of constraints becomes more complex when design variables and constraints contain uncertainties. That is, there are cases where some design points in the variable region that satisfy the constraints and some that fails. To minimize a function with variations in constraints, Parkinson et. al. (1990) advocate a two step solution method. The first step addresses the optimization problem with only the nominal constraints and variation (Æb) and does not include the function variations. The second step involves optimizing the problem with constant constraint variation built into the model. A major limitation in this two-step procedure is that one can not study the effect of variations in individual variables on constraints. In statistical optimization, one should be interested not only in minimizing the effect of variations (tolerances) in design variables on performance, but also in controlling the effect of variations in design variables on the active constraints.

Many engineering problems are too complex to be simplified into a set of equations or modeled with numerical simulation programs. Distortion due to heat treatment is a good example. For such a case, one needs to resort to experimental design to reduce the number of experiments in searching robust optimum. Taguchi proposed inner and outer orthogonal arrays for experimental design [Byrne et al. 1987], which incurs critiques from statistics experts. Taguchi uses the inner array for controllable factors and the outer array for noise factors, which is often unnecessary and results in a large number of experiments [Montgomery 1991]. His experimental design scheme also falls short in explaining the alias structure and may lead to the incorrect conclusion when significant interaction is present.

This paper describes research at Ohio State on robust design optimization using fractional factorial experiments (FFE) and simulation. The proposed method incorporates manufacturing and operational variances to achieve designs with robust and optimal performance. The procedure optimizes the expected value of a performance characteristic and its variation subject to a set of constraints. To account for variations on design variables and parameters, we use the concepts from statistical design of experiments to approximate the performance characteristic. The paper describes our approximation that adapts Gauss-Hermite Quadrature Integration [D'Errico et al., 1988] to fractional factorial design to estimate expected value and its deviation index. The procedure applies to performance simulation programs based on both physical models and experiment-based models.

Further, the paper addresses variations in constraints due to uncertainty in design variables. When design variables and/or constraint equations vary, a candidate design that may violate a constraint even if the target values satisfy the constraints depending on the variations. The paper extends the notion of constraint activity to incorporate various cases of constraint satisfaction. Based on these definitions, we propose the following methods to incorporate constraint variations in robust design optimization: 1) Method using heuristics that evaluate constraints at the worst combinations of design variables, 2) Method with built-in constraint variation that models constraints using first order Taylor expansion and 3) Method based on differentiating KKT optimality conditions.

The design of spur and helical gears with minimum transmission error serves as an example of using simulation program for parameter design. We adapt conventional optimization algorithms and use the proposed methods for approximating the expected objective function value and evaluation of uncertain constraints. The key issue in gear design is to determine the optimal combination of geometric design variables like number of teeth, pressure angle that minimizes transmission error subject to constraints like minimum number of teeth to avoid undercut and maximum bending stress. The manufacturing errors on gear profile modification and assembly misalignments also introduce the variations in design variables. The performance characteristic of interest was transmission error which in turn reflects the noise and vibration level of meshing gears.

We also applied our method to heat treated parts. Heat treatment is a common practice to improve material properties such as surface hardness, residual stress, mechanical strength, and machinability. However, dimensional distortions and cracks usually accompany heat treatment unless conditions are closely controlled. Correcting the geometric distortions due to heat treatments by straightening or machining is quite costly. Many studies address process control to improve dimensional stability during heat treatment. However, few explore the possibility of containing distortion by affecting the part design. The objective of our research is to find a robust part geometry that is insensitive to material and process variations. Here, we use an experimental model rather an analytical model.

2. ROBUST OPTIMIZATION

In conventional optimization problems, the objective is to optimize a linear or a non-linear function of many variables subject to a set of constraints. Due to manufacturing and operation variations, the control variables and the objective function will have statistical distributions. Probabilistic optimization aims to optimize the expected value of the objective function subject to constraints with uncertainties [Siddall 1984]. The concept is formulated as follows.

Minimize $$E[Y(X)] = \int_{X^-}^{X^+} Y(X) \cdot P(X) \cdot dX \qquad (1)$$

Subject to $X^- \le X \le X^+$ and $X=[x_1, x_2, \ldots x_k]^T$

$\mathrm{Prob}[g_1(X, b_1) \le 0 \cap g_2(X, b_2) \le 0 \cap \ldots \cap g_J(X, b_J) \le 0] \ge D$

$E[h_n(X)] = 0$ for $n=1,2,\ldots,N$

where $P(X)$ is the joint probability function of X and D is the required probability of a feasible design

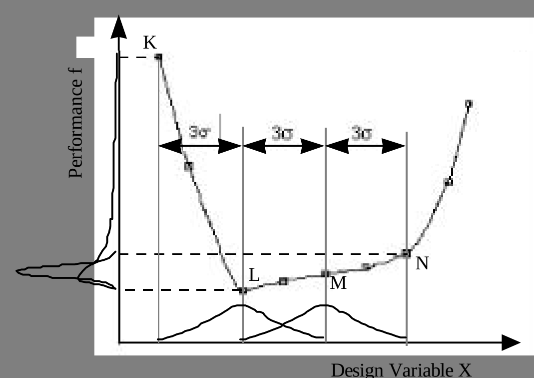


FIGURE 2. PEAK OPTIMUM *L* AND ROBUST OPTIMUM *M*

However, in most engineering problems, the joint probability function of design variable $\{x_1, x_2, ...x_k\}$ is unknown. The computation of expected values and variances using equation (1) will be expensive, even when the joint probability function is available. One needs a method to approximate equation (2) by an expression that will be easier to evaluate when using performance simulation programs or actual experiments. Yu et al. (1993) integrated Gauss-Hermite quadrature integration and fractional factorial experiment to estimate the expected value Y_e and the deviation index *(DI).*

$$Y_e = E[Y(X)] \approx \sum_{i=1}^{N} W_i \cdot Y_i \quad (3)$$

$$DI = \sqrt{\sum_{i=1}^{N} W_i \cdot (Y_i - E(Y))^2} \quad (4)$$

where W_i is the combined weights at factorial points as shown in figure 3.

The method gives superior approximations in cases of models with significant interaction and nonlinearity effects. The objective function *F(X)* (Eq. 2) during robust optimization is approximated as follows.

$$F(X) = Y_e + b * DI \tag{5}$$

Another challenge in robust design is handling constraints. If the constraints involve variations, one must expand the definition of constraint activity, as illustrated in Figure 4. In our work, we seek the case with statistical active constraints. We have devised several methods to find solutions that are statistical actively constrained, each of which can easily be integrated into non-linear programming methods such as BFGS [Vanderplaats, 1984].

1) **<u>Method 1: Using heuristics</u>.** This method evaluates the objective function and constraints at the worst combinations of design variables. A constraint *($g_j(X)$ - b_j ² 0)* during peak optimization would be modeled during statistical optimization as

$$Max\left\{\left[g_j(\mathbf{y}) - b_j\right], \forall \mathbf{y} \in W(\mathbf{x})\right\} \leq 0 \tag{6}$$

2) **<u>Method 2: Constraints with Built-in Constraint Variation</u>.** Instead of using FFE to determine which worst combination of design variables has a larger constraint value, this method uses the gradient information to evaluate the worst possible value of the constraint. We modify the feasible region to account for the maximum possible variation in constraint g due to deviation Δx_i .

$$g_j(X, b_j) + \sum_{i=1}^{n} \left| \frac{¶g_j}{¶x_i} \Delta x_i \right| \leq 0 \tag{7}$$

3) **<u>Method 3: Approach using KKT Conditions</u>.** This method is based on differentiating KKT conditions of optimality with respect to design variables. The first step involves finding the robust optimum without considering any variation in the constraints. Due to inherent variations in the constraints, the optimum achieved at this stage may be infeasible. The second step involves finding the change in the optimum design variables for a change in the constraints. At this stage, the following Lagrangian equation gives the Lagrange multipliers λ^* and the constrained optimum X^*.

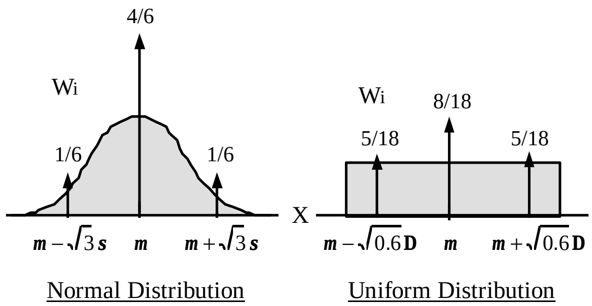
$$L(X, l, b) = F(X) + \sum_{j=1}^{m} l_j \, g_j(X, b_j) \tag{8}$$

The change of the λ^* and X^* due to the variation of the constraints are

$$\Delta\, l_j = -\sum_{k=1}^{m} \frac{¶\, l_j^*}{¶b_k} \Delta g_k \qquad \Delta x_j = -\sum_{k=1}^{m} \frac{¶x_j^*}{¶\, b_k} \Delta g_k \tag{9}$$

Hence, the statistically active optimum is given by

$$X_{new}^* = X_{old}^* + \Delta X \tag{10}$$



σ: Standard deviation of the normal variable
2^2: Tolerance range of the uniform distribution

FIGURE 3. SELECTION OF LEVELS AND WEIGHTING OF NORMAL AND UNIFORM VARIABLES

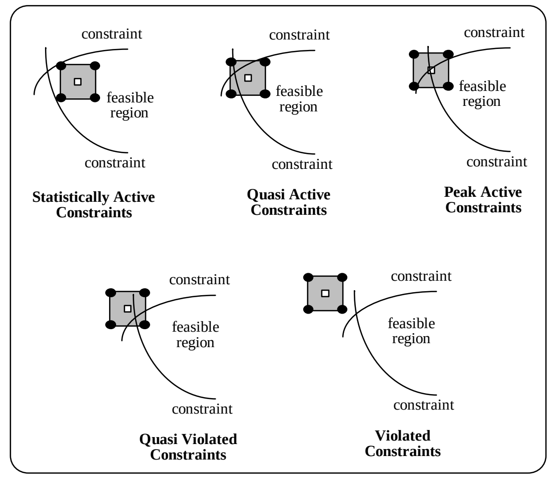


FIGURE 4. CRITICALITY OF CONSTRAINTS

Sundaresan et al (1993) documents the details of these techniques. The first method based on brute force with heuristics and the second method based on first order approximation of the constraint variation are ideal for problems where evaluation of the constraints is easy and not time-consuming. The third method, based on differentiating KKT conditions, is suitable for problems where constraint evaluation is computationally expensive. However, the third method is a two-step procedure where the method assumes that there is no change in the set of active constraints during the second step. This problem, however, does not occur in the first two methods. In cases where the evaluation of constraints involve analysis procedures (like FEM) that are time consuming, it is recommended that one should come up with simple algebraic equations by using regression analysis. Using curve-fitted equations significantly reduces computing time.

3. GEAR EXAMPLES

We applied our unconstrained robust optimization method using equation (1) to profile modification (figure 5) of helical gears [Sundaresan 1989]. The objective function was peak to peak transmission error which reflects the noise and vibration characteristics. That is, we sought a gear pair that were quiet and also insensitive to manufacturing errors. Figure 6 shows that our statistical optimum, although slightly inferior in the target value, is significantly lower in expected value and has a narrower variation range.

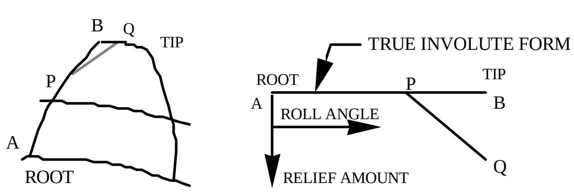


FIGURE 5. PROFILE MODIFICATION

TABLE 1. RESULTS OF VARIOUS METHODS

METHODS USED	FUNC-TION	N_p	ϕ_n	# FE	# GE
Peak Optimum (Pt. P)	53.416	58.21	11.36	188	188
Method 1	53.726	57.82	11.74	193	579
Method 2	53.662	58.00	11.68	196	579
Method 3	53.735	57.84	11.75	189	191

We also tackled a constrained design problem (spur gear) using the methods outlined above. Figure 7 shows a contour plot of peak to peak transmission error as a function of Number of Teeth and Pressure Angle. The plot also shows constraints related to undercut and bending stress. Table 1 shows that our method successfully finds the robust optimum with comparison of different methods.

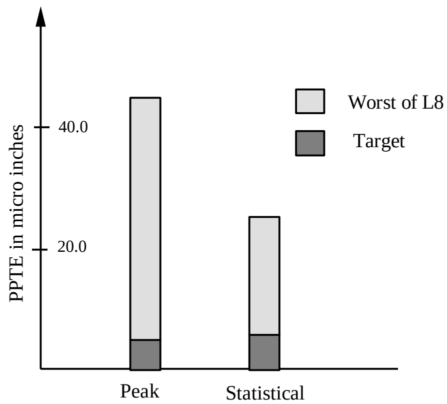


FIGURE 6. OPTIMIZATION RESULTS

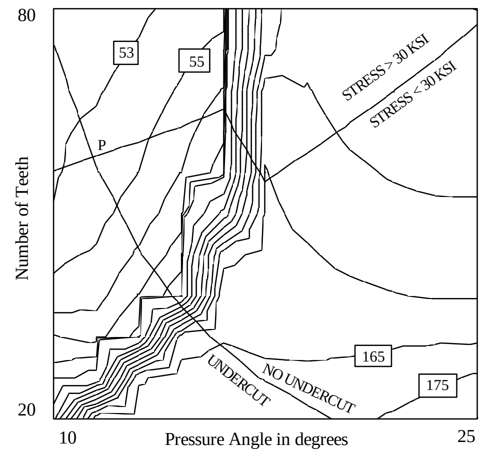


FIG. 7 CONSTRAINTS AND FUNCTION CONTOURS

4. ROBUST DESIGN USING FACTORIAL EXPERIMENTS

The previous sections focused on problems for which we had a performance simulation program, and we assumed that the effect of variations on performance are monotonic within its range. In many cases, this is not true, and the problem may also involve significant non-linearity within the range of variations. One can use 3-level FFE to evaluate the distribution parameters of each design point for a second order model, but higher level FFEs result in complex experiments. We adopt the two-level fractional factorial experiments augmented with center points (2FFEC), because of its efficiency and its immunity to nonlinearity of responses. The augmentation of center points enables us to identify the pure quadratic effects of the model. If the non-monotonic effect is significant, one needs to include the quadratic regressors in the model.

FFE always leads to the confounding of main effects with interaction effects. Even if the other confounded interactions are negligible, we still can obtain satisfactory estimates of the interesting effects. Each 2^{k-p} fractional factorial design can be characterized by its Defining Relation, I, which determines the confounding structure of the design. Since all these aliases are lumped together, the estimates for a certain factor will actually stand for a linear combination of all the aliases of that factor. In practice, interactions higher than two factors are often negligible. In this case, the confounding structure will be greatly simplified.

We extend the experiments sequentially based on the complexity of quadratic effects of the model. The number of parameters one can estimate from the $2^{(k-p)}+1$ factorial experiment is $2^{(k-p)}$. If the number of parameters exceeds the degree of freedom (DOF) of the experiment, the alternate fraction with defining relation -I is augmented to expand the design to a $2^{(k-p+1)}+1$ factorial experiment. If the DOF of a full factorial, 2^k+1, is still inadequate, one can construct a central composite design by adding 2k axial points, $(\pm 2^{k/4}, 0, 0, ..., 0)$, $(0, \pm 2^{k/4}, 0, ..., 0)$, ..., and $(0, 0, ..., \pm 2^{k/4})$ to the 2FEC design to fit a complex second order model [Montgomery 1991]. Using a statistics analysis program, such as SAS or JMP [JMP, 1989] will easily produce us with the estimates of the effects of the second order regression model .

5. ROBUST DESIGN FOR HEAT TREATED PARTS

Our current work applies our experimental design and optimization scheme to the robust geometry design for heat treated parts. The goal is 1) to determine the non-critical dimensions that leads to the minimum variance on the distribution of critical dimensions after heat treatment, and 2) to specify green part dimension to compensate for the size change such that the dimensions after heat treatment will fall mostly inside the tolerance range. Since a theoretical model is not available, we must resort to our experimental design scheme.

Figure 8 shows an example shaft and the distribution of the pin diameter before and after heat treatment. The dimensional specification of the spline is fixed due to functional requirements, while the diameter of the bearing section, D_b, has some adjustable design range. Our example focuses on investigating the relationship between the outboard

must apply
face hardening
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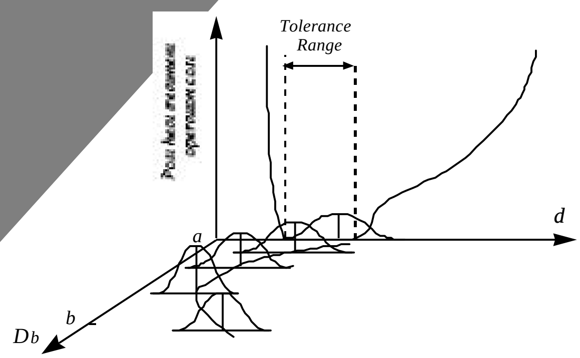

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FIGURE 9. MODIFICATION OF GREEN PART DIMENSION

6. CONCLUSION

This paper proposed a systematic procedure that provides a robust optimum for designs that involve either computer simulations or actual experiments. The salient features of this methodology include

1) **Use of Experimental Design**: The statistical optimization technique developed in this research applies statistical design of experiments concepts in the computation of the expected value and its variation of a performance function. The proposed estimation scheme applies the modified 3-point Gauss quadrature integration to 2FFEC to assure superior accuracy for systems with significant nonlinear and interaction effects.
2) **Objective Function**: The paper defined a new objective function that consists of a weighted sum of expected mean and deviation index (DI) for statistical optimization. The new definition is more versatile and meaningful than our previous function which used a sensitivity index [Sundaresan 1991].
3) **Constrained optimization**: This research proposed three techniques to solve constrained statistical optimization problems and defined various conditions of criticality of a constraint. The three statistical optimization techniques not only incorporate the effect of variations in design variables(ΔX_i), but also the uncertainty in the constraints(Δg_i) due to variations in design variable.

Our current investigation focuses on the interaction and dependencies among the variations in design variables and constraints. For example, dimensional variations of automotive transmission shafts under heat treatment process are strongly coupled. Also, in injection molded plastic gears, profile errors do not vary independently, because volumetric shrinkage affect several design variations. The incorporation of interaction in variation poses a significant challenge.

ACKNOWLEDGMENTS

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